Re-Exam

Autumn 2019

Important: Please make sure that you answer all questions and that you properly explain your answers. For each step write the general formula (where relevant) and explain what you do. Not only the numerical answer. If you make a calculation mistake in one of the earlier sub-questions, you can only get points for the following subquestions if the formula and the explanations are correct!

1. Short questions.

(a) "When applying iterated elimination of *weakly* dominated strategies, it is possible to eliminate Pure Strategy Nash Equilibria." 1) State if this is true or false 2) Provide a small example proving your point.

Solution: This is true.

		P2 L	P2 R
P1	U	$3,2$	$0,0$
P1	D	$3,0$	$1,2$

NE = (U, L), (D, R)

In a NE where player 1 is indifferent between the NE-payoff and her payoff from deviating (3 when playing U or D as response to L), the NE-strategy can be weakly dominated if player 1's alternative strategy would give a higher payoff in the case where player 2 deviates from his NE strategy as well (P1 plays D and P2 plays R). Thus strategy U could be eliminated and with it the NE (U.L).

(b) "In a mixed strategy NE players must be indifferent between their respective pure strategies." True or false? Give a small example and explain in 2-3 sentences.

Solution: Suppose it were not true. Then there must be at least one pure strategy s_i that is assigned positive probability by my best-response mix and that yields a lower expected payoff against the strategy of the other player. If there is more than one, focus on the one that yields the lowest expected payoff. Suppose I drop that (low-yield) pure strategy from my mix, assigning the weight I used to give it to one of the other (higheryield) strategies in the mix. This must raise my expected payoff (just as dropping the player with the lowest batting average on a team must raise the team average). But then the original mixed strategy cannot have been a best response: it does not do as well as the new mixed strategy. This is a contradiction.

(c) In a family, there are 3 sisters, named Alice, Beatrice, and Caroline. They have a big piece of cake, whose size we normalize to 1. Alice divides the cake into 3 pieces, which can be of different sizes. Next, Beatrice picks one of the pieces for herself. Next, Caroline picks one of the remaining two pieces. Alice picks the last piece. Each sister's payoff is the size of the cake she gets. a) Using backward induction, show how she will devide the cake. Explain your steps, the pure outcome is not enough. b) Repeat (a) assuming instead that Beatrice picks a piece for Alice; Caroline picks a piece for Beatrice, and the remaining piece is given to Caroline. Explain your steps.

Solution: a) Given the remaining two pieces, Caroline chooses the larger one, leaving the smaller one to Alice. When Beatrice's turn, she chooses the largest of the three pieces. Therefore, given any division (x_1, x_2, x_3), Beatrice gets the largest, Caroline gets the middle one, and Alice gets the smallest. The unique solution is (1/3, 1/3, 1/3). b) One solution is as follows: Given the remaining two pieces, Caroline gives the smaller to Beatrice, keeping the larger one for herself. Given the three pieces, Beatrice will get the smaller of the remaining two. Hence, it is a best response to Beatrice to give Alice the smallest of the three. Therefore, once again Alice receives the smallest of the three pieces, and therefore she divides the cake equally (1/3, 1/3, 1/3).

(d) Explain what is meant by the term common values and distinguish this from private values. Explain why the winner's curse arises with common values but not with private values.

Solution: In common value auctions the value of the item for sale is identical amongst bidders, but bidders have different information about the item's value. This stands in contrast to a private value auction where each bidder's private valuation of the item is different and independent of peers' valuations. In a common value auction you might overestimate the value of the good based on the signal you have received. If you win, you likely bid too much. In a private value auction you should never bid more than your valuation. Winner's curse means that you bid to much. If you win this is information that the other people had negative information about the value of the good.

2. Two city farmers, Mia and Iben, let their chickens run around on their rooftop garden. They can choose to use the common resource lightly or heavily and the resulting strategic interaction may be described as a simultaneous-move game. The payoff matrix is the following:

		Iben: Light	Iben: Heavy
Mia	Light	40, 40	20, 55
	Heavy	55, 20	30, 30

(a) Find the Nash equilibrium of the game and show that it is an example of "Prisoners' Dilemma" games.

Solution: The Nash equilibrium is heavy, heavy with a payoff of 30 for both players. Indeed, both Mia and Iben have a dominant strategy to use the common land heavily. In addition, the Nash equilibrium is Pareto dominated by the outcome 40,40 arising when both players choose the dominated strategy light. The two features are characteristic of Prisoners' Dilemma games. The game describes the so-called "tragedy of commons" in which the users of a common resource have an incentive to over-use it.

(b) Suppose that the same game is repeated infinitely. Is the (light, light) outcome a SPNE if both players play a trigger strategy and have a discount factor of 0.7?

Solution: When the game is repeated infinitely the trigger strategy is the following:

- Start playing cooperatively (light)
- Play cooperatively as long as the other player chooses light
- Whenever the other player chooses heavy, switch to heavy and play it forever.

The couple of strategies (light, light) can be sustained as SPNE when both players play a trigger strategy and if they care enough about future payoffs; i.e., if the discount factor δ is high enough. The net present value of cooperation is larger than the net present value of deviation. If Mia always plays light against Iben who follows a trigger strategy, Mia's payoff will be 40 forever. The present value of cooperation is then the present value of this infinite sequence; i.e.,

$$\Pi_coop = 40 + \delta * 40 + \delta^2 * 40 + \delta^3 * 40 + ... = \frac{40}{1-\delta} \tag{0.1}$$

By deviating from cooperation and playing heavy, Mia will get 55 in the first period but will face the reaction of her opponent. From the following period Iben will always play heavy. So, after her own deviation, the best option for Mia is to stick with heavy forever. The net present value of this deviating strategy is then:

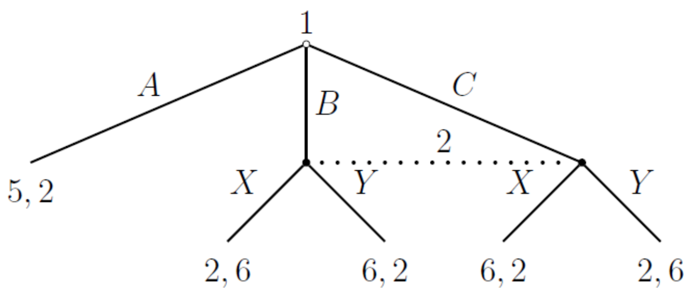
$$\Pi_dev = 55+\delta*30+\delta^2*30+... = 55+\delta(30+\delta*30+\delta^2*30+...) = 55+\delta*\frac{30}{1-\delta} \tag{0.2}$$

Playing cooperatively against a player adopting a trigger strategy is sustainable as a Nash equilibrium from Mia's viewpoint only if $\Pi_coop > \Pi_dev$

$$\frac{40}{1-\delta} > 55 + \delta * \frac{30}{1-\delta} \rightarrow \delta > \delta* = \frac{15}{25} = 0.6 \tag{0.3}$$

Since the game is symmetric, this condition holds for both Mia and Iben. Given that their discount factor is $0.7 > \delta*$, the cooperative outcome (light, light) is sustainable as a SPNE of the infinitely repeated game when both players play the trigger strategy. This threat of punishment, therefore, may represent a solution to the tragedy of commons.

3. Have a look at the following game:



(a) Is it a dynamic or static game? How many proper subgames are there?

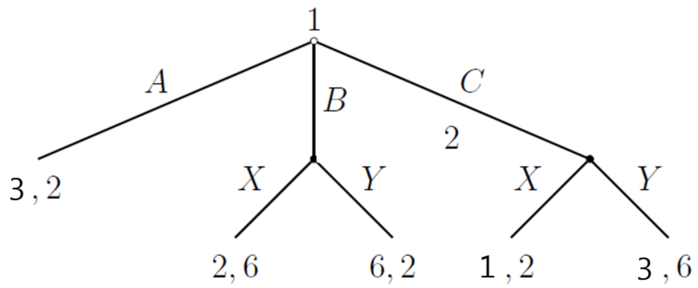
Solution: It is a dynamic game. There are no proper subgames.

(b) Which solution concept is appropriate to apply in this game and why? What does the equilibrium look like? (Hint: Describe the reasoning of each of the players).

Solution: Perfect Baysian Equilibrium because it discards equilibira that are not sequentially rational. In the PBE Player 1 will play A and Player 2 will have beliefs close to p = 1/2 and will randomize between playing X and Y. If Player 2 gets to move, he would want to play X if he thinks player 1 played B and play Y if he thinks player 1 played C. If Player 2 chooses X, Player 1 will want to play C rather than B and vice versa.

(c) Suppose that we delete the information set between player 2's decision nodes. This means 2 can observe whether 1 chose B or C. What solution concept should we apply now? Find the unique equilibrium under that concept.

Solution: Now we should apply Subgame perfect nash equilibirum. This leads to (A, XY).



(d) Now consider the new payoff version of the game. Find all pure strategy nash equilibria of this game.

Solution: The PSNE are (A,XX), (A,XY) and (C,XY).

(e) Which of these are subgame perfect?

Solution: (A,XY) and (C,XY) are subgame perfect.

4. Bose and Soundbox are competing for customers in Copenhagen who like to listen to good music. The demand for Bose's products are given by $q_1(p_1, p_2) = 11 - p_1 + 0.5p_2$. The demand for Soundbox products are given by $q_2(p_1, p_2) = 11 - p_2 + 0.5p_1$. Both speaker companies face a cost of supplying their products that amounts to $C(q) = 4q$.

(a) Compute the SPNE when both companies simultaneously set their prices. Calculate the resulting profits.

Solution: Bose solves the following problem
$maxp_1(11 - p_1 + 0.5p_2)(p_1 - 4)$
From the FOC we obtain the reaction function:

$$p_1 = \frac{15 + 0,5p_2}{2} \tag{0.4}$$

For Soundbox we get:

$$p_2 = \frac{15 + 0,5p_1}{2} \tag{0.5}$$

This leads to $p_1 = p_2 = 10$
Substituting in the profits functions:

$$\Pi_1 = \Pi_2 = 36 \tag{0.6}$$

(b) Suppose now that, before the choice of prices, Bose can develop a new technology at the cost of 10 units, which will reduce its marginal cost to zero. However, Soundbox will immediately receive the new technology paying a fixed cost of 5 (simultaneaous game, no decision necessary). Calculate the SPNE of the given game. Will Bose develop the new technology?

Solution: If Bose does not develop the technology, the game in a) will follow. If Bose develops the technology, it will solve the following problem:

$$max_p 1(11 - p_1 + 0.5p_2)p_1 \tag{0.7}$$

From the FOC we obtain the reaction function:

$$p_1 = \frac{11 + 0,5p_2}{2} and p_2 = \frac{11 + 0,5p_1}{2} \tag{0.8}$$

Solving the system formed by the two reaction functions we get

$$p_1 = p_2 = \frac{22}{3} = 7,33 \tag{0.9}$$

Substituting in the profits functions (do not forget to substract the fixed costs):

$$\Pi_1 = 43,72 \Pi_2 = 48,72 \tag{0.10}$$

Despite the fact that Soundbox copies the technology and gets higher profits, Bose is still better off if it develops the technology. The Nash equilibrium is: (Develop the technology, $p_1 = 7,33$, $p_2 = 7,33$/ Don't develop $p_1 = 10, p_2 = 10$)

(c) Explain intuitively what would happen to prices and to profits if Bose and Soundbox would be selling non-homogeneous goods and if Bose would increase its prices.

Solution: When each firm produces a differentiated product, its demand doesn't become zero when it raises its price. The Bertrand model concludes that if one firm increases it price, the other firms in a differentiated oligopoly should also increase theirs because this will increase its profit. So if Bose increases its prices, Soundbox should do the same and profit would increase.